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A PERTURBATION THEORY OF TWO-DIMENSIONAL TRANSONIC WIND TUNNEL --ETC(U)
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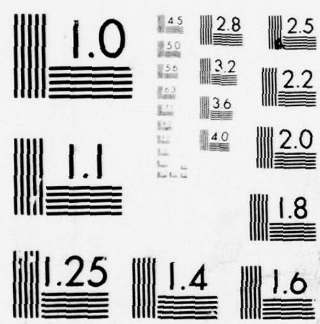

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LEVEL II

A PERTURBATION THEORY OF TWO-DIMENSIONAL TRANSONIC WIND TUNNEL WALL INTERFERENCE

by

Y. Y. Chan

National Aeronautical Establishment

OTTAWA

APRIL 1979

NRC NO. 17476

AERONAUTICAL REPORT LR-598

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The wind tunnel wall interference in transonic speed is formulated as a perturbation to the basic flow around the airfoil in free air. The perturbation equation is derived from the transonic small disturbance equation and is linear but with variable coefficients containing the non-linear solution of the basic flow. The equation is solved numerically by a direct matrix method using the classical boundary condition for a porous wall. The solution in terms of lift versus angle of attack agrees well with that calculated directly from the small disturbance equation.

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A PERTURBATION THEORY OF TWO-DIMENSIONAL TRANSONIC WIND TUNNEL WALL INTERFERENCE

(THÉORIE DE LA PERTURBATION DE L'INTERFÉRENCE DE PAROIS DANS UNE SOUFFLERIE BI-DIMENSIONELLE TRANSSONIQUE),

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SUMMARY

The wind tunnel wall interference in transonic speed is formulated as a perturbation to the basic flow around the airfoil in free air. The perturbation equation is derived from the transonic small disturbance equation and is linear but with variable coefficients containing the non-linear solution of the basic flow. The equation is solved numerically by a direct matrix method using the classical boundary condition for a porous wall. The solution in terms of lift versus angle of attack agrees well with that calculated directly from the small disturbance equation.

RÉSUMÉ

L'interférence que causent les parois d'une soufflerie transsonique est exprimée comme une perturbation agissant sur l'écoulement autour de la surface portante dans l'air libre. L'équation de perturbation est établie à partir de l'équation des petites perturbations aux vitesses transsoniques; elle est linéaire mais comprend des coefficients variables contenant la solution non linéaire de l'écoulement primaire. L'équation se résout numériquement au moyen d'une méthode matricielle directe, en utilisant la condition limite classique pour une paroi perforée. La solution exprimée en termes de la variation de portance en fonction de l'angle d'attaque concorde bien avec celle calculée directement à partir de l'équation de faible perturbation.

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A PERTURBATION THEORY OF TWO-DIMENSIONAL TRANSONIC WIND TUNNEL WALL INTERFERENCE

1. INTRODUCTION

Interference of the wind tunnel wall on the flow around a model is an important and practical subject in aerodynamics of simulation. Through the years a large amount of effort has been devoted to the problem. An excellent summary of the results of these efforts can be found in the review by Garner et. al. (Ref. 1). The classical approach to the problem is to consider the wall interference as a perturbation to the basic flow past the model in free air. The flow induced by the perturbation is then treated as *corrections* to the measured results of the model. This method works well for subsonic flow because the linear nature of the flow regime warrants the principle of superposition of solutions.

In the transonic range, the governing equation for the flow past a model is non-linear. The classical method, in principle, can no longer be applied and elaborate calculations of the flows bounded by tunnel walls because necessary (Refs. 2, 3, 4). These rigorous methods usually involved numerical solutions, require a large amount of computation and thus are not favourable for practical applications. The subsonic theory has also been applied in the transonic range (Ref. 5) based on the fact that the far field structures for the subsonic and the transonic flows are highly similar. However, whether the subsonic linear solution can be applied all the way to the airfoil around which the flow is definitely non-linear, has not been proven.

The simplicity of the classical method, on the other hand, is attractive to practical engineering. It would be desirable if a method could be derived with the preservation of the simplicity of the classical method and at the same time being correctly formulated for the transonic range. The present report is a first attempt in this direction.

In keeping the simplicity of the classical method, the present approach considers that the wall interference is a small perturbation to the basic flow around the airfoil. However, the basic flow is now transonic and non-linear. Within the framework of transonic small disturbance theory, the perturbation equation is then derived from the transonic small disturbance equation. The resulting equation is now linear because of the small perturbation assumption but with variable coefficients containing the non-linear solution of the basic flow. This equation is then solved, satisfying the imposed boundary conditions at the tunnel wall and the airfoil. The solution thus gives the first order correction to the basic flow due to the presence of wind tunnel walls. The results of the present method compare favourably with those directly calculated from the small disturbance equation (Refs. 2, 4). This comes as no surprise since the success of applying perturbation techniques to the transonic non-linear basic flow has been demonstrated by Cheng and Hafez (Ref. 6) for three-dimensional flows and Nixon (Ref. 7) for two dimensional cases.

The present method is simple to use for practical wall interference calculations as the linear equation can be solved by a standard routine with a short computing time. This is particularly important if parameters such as tunnel wall porosities have to be determined by matching the solution with the measured wall pressure distributions (Ref. 5). For this case, the non-linear equation of the free air basic flow will be solved only once, and the searching routine involves solving the linear equation only.

The present calculation also shows that the angle of attack correction obtained by the subsonic linear theory applied in the transonic range, is surprisingly close to that of the present results. The Mach number correction from the linear theory however, does not compare so favourably. This is attributed to the fact that in the linear approach, the non-linear contribution to the doublet strength, which is strongest near the airfoil, is completely neglected.

2. GOVERNING EQUATIONS AND CONDITIONS

The problem considered is a transonic inviscid flow past an airfoil model situated in a wind tunnel (Fig. 1). The airfoil has a chord of unity and a thickness δ. The tunnel walls are located at a distance H above and below the model (the model need not be located at the middle of the tunnel; the central position is chosen for simplicity). The airfoil is assumed to be thin and the angle of attack small. Within the framework of the transonic small disturbance theory (Refs. 8, 2), the equation governing the flow inside the tunnel is thus

$$[K - (\gamma + 1)\,\phi_x]\,\phi_{xx} + \phi_{yy} = 0 \tag{1}$$

K is the transonic similarity parameter defined as

$$K = \frac{1 - M^2}{M^{2m}\ \delta^{2/3}} \tag{2}$$

where δ is the thickness of the airfoil and M the free stream Mach number. The velocity potential ϕ and the co-ordinate normal to the flow direction y have been scaled according to the similarity transformation.

$$\begin{aligned} \phi &= \delta^{-2/3}\ M^n\ \phi^* \\ y &= \delta^{1/3}\ M^m\ y^* \end{aligned} \tag{3}$$

where the started variables denote their corresponding quantities in the physical plane.

The boundary condition on the airfoil is that the flow should be tangent to the surface,

$$(\phi_y)_{u,\ell} = M^{n-m} \left[\left(\frac{\partial F}{\partial X} \right)_{u,\ell} - \frac{\alpha}{\delta} \right] \quad \text{at } y = \pm 0 \text{ on airfoil} \tag{4}$$

where F is the profile of the fairing and α the angle of attack. The tunnel wall is perforated for flow ventilation as required for the transonic tests. The boundary condition is given as (Refs. 1, 2)

$$\phi_x \pm \frac{1}{P}\,\phi_y = 0 \quad \text{at } y = \pm H \tag{5}$$

where P is the porosity of the tunnel wall and H the distance between the airfoil and the wall. These parameters are also scaled as follows:

$$\begin{aligned} H &= \delta^{1/3}\ M^m\ H^* \\ P &= \delta^{1/3}\ M^{-m}\ P^* \end{aligned} \tag{6}$$

The exponents m and n in the scaling are set to be (Refs. 2, 9)

$$m = 1/2, \quad n = 3/4$$

If there is shock wave occurring in the flow field, the weak solution should satisfy the discontinuity condition across the shock (Refs. 8, 10)

$$\langle K - (\gamma + 1)\,\phi_x \rangle [\![\phi_x]\!]^2 + [\![\phi_y]\!]^2 = 0$$
$$[\![\phi]\!] = 0 \tag{7}$$

where the shock shape is expressed as

$$x = x^D(y) \tag{8}$$

The symbol $[\![\ ]\!]$ denotes the difference and $\langle \ \rangle$ the arithmetical mean across the shock.

3. PERTURBATION EQUATIONS AND CONDITIONS

In the present formulation, the basic flow field is that of an airfoil in the free air. The restriction of the tunnel walls is considered as small perturbation for the basic flow. Thus the velocity potential can be written in two parts, the free air potential and the interference potential

$$\phi = \phi_o + \phi_1 \tag{9}$$

where ϕ_1 is one order of magnitude higher than ϕ_o. From the problem defined in the last section, the equations and the conditions for the free air and the interference flows can be derived.

Free air flow

$$[K - (\gamma + 1)\,\phi_{ox}]\ \phi_{oxx} + \phi_{oyy} = 0 \tag{10a}$$

$$(\phi_{oy})_{u,\ell} = M^{n-m}\left[\left(\frac{\partial F}{\partial X}\right)_{u,\ell} - \frac{\alpha}{\delta}\right] \quad \text{at } y = +0 \text{ on airfoil} \tag{10b}$$

$$\phi_{ox}, \phi_{oy} \to 0 \qquad \text{as } x^2 + y^2 \to \infty \tag{10c}$$

and

$$\langle K + (\gamma + 1)\,\phi_{ox} \rangle [\![\phi_{ox}]\!]^2 + [\![\phi_{oy}]\!]^2 = 0 \tag{10d}$$

$$[\![\phi_o]\!] = 0 \tag{10e}$$

Interference flow

$$[K - (\gamma + 1)\,\phi_{ox}]\,\phi_{1xx} - (\gamma + 1)\,\phi_{oxx}\,\phi_{1x} + \phi_{1yy} = 0 \tag{11a}$$

$$\phi_{1y} = 0 \qquad \text{at } y = \pm 0 \text{ on airfoil} \tag{11b}$$

$$\phi_{1x} \pm \frac{1}{P}\phi_{1y} = -\left(\phi_{ox} \pm \frac{1}{P}\phi_{oy}\right) \qquad \text{at } y = \pm H \tag{11c}$$

The shock condition required further consideration because the shock location may be shifted. The shock location can be written as

$$x^D = x_o^D(y) + x_1^D(y)$$

and the shock conditions should be satisfied along x^D. However, calculation of the first order equation, Equation (11) depends on the zero order solution and its shock location. Thus the shock condition for the perturbation equation has to be transferred to the unperturbed boundary by analytic continuations (Ref. 10). The shock conditions for the interference flow is therefore

$$<(\gamma+1)\,[\phi_{1x}\,(x_o,y)+\phi_{oxx}\,(x_o,y)\;x_1{}^D(y)]> \;=\; 2\,x_{oy}^D\,x_{1y}^D \tag{12a}$$

$$[\![\phi_1\,(x_o,y)\;+\;\phi_{ox}\,(x_o,y)\;x_1{}^D(y)]\!] \;=\; 0 \tag{12b}$$

Far Field Boundary Conditions

In computing the free air solution, the far field boundary condition has to be specified. If the computed flow field does not extend to infinity, the asymptotic solution of the transonic small disturbance equation, Equation (10) can be used (Refs. 2, 9)

$$\phi_o(x,y) \sim -\frac{\Gamma}{2\pi}\tan^{-1}\frac{K^{1/2}y}{x} + \frac{D}{2\pi K^{1/2}}\frac{x}{x^2+y^2} \tag{13}$$

where the doublet strength D is given as

$$D = \frac{1}{\delta}\int_{-0.5}^{0.5}(F_u - F_\ell)\,dx + \frac{\gamma+1}{4}\int\!\!\int_{-\infty}^{+\infty}\phi_x{}^2\,dxdy \tag{14}$$

and Γ is the circulation scaled as

$$\Gamma = \delta^{2/3}\,M^n\,\Gamma^*$$

By satisfying the Kutta condition, the circulation Γ is equal to the potential difference across the airfoil plane at the trailing edge. These conditions should be incorporated with the zero order equation and the other conditions given in Equation (10) to complete the formulation for the free air case.

For the interference potential, the conditions at far upstream and downstream have to be specified. The asymptotic solutions for this case are given by Murman (Ref. 2) and Catherall (Ref. 4). If the flow field extended to infinity at both ends, then the condition can be written up to the first order as

$$\phi_1 = -\phi_o \qquad \text{at } x \to \pm\infty \tag{15}$$

where ϕ_o is given by Equation (13) at the limit of x approaching infinity. The potential jump at the trailing edge will give the change of circulation due to the wall interference

$$\Delta\Gamma = \phi_1{}^+ - \phi_1{}^- \qquad \text{at the trailing edge} \tag{16}$$

The formulation for the interference potential is thus complete.

4. INTERFERENCE CORRECTIONS

In the classical method of the tunnel wall interference (Ref. 1) the interference potential is calculated without enforcing the boundary condition on the airfoil surface by considering that the airfoil shrinks to a point singularity. The correction is obtained by evaluating the velocity component at the location of the singularity. The streamwise component is considered as the change of the free stream velocity (or Mach number) and the normal component the change of angle of attack. These can be written as follows (Ref. 1)

$$\Delta M = M\left(1+\frac{\gamma+1}{2}M^2\right)\delta^{2/3}\,M^{-n}\,\phi_{1x} \tag{17}$$

$$\Delta\alpha = \delta M^{m-n} \phi_{1y} \tag{18}$$

This approach is justified if the tunnel height 2H is much larger than the chord of the model. Thus the perturbation parameter ϵ is of the form 1/H and that $\epsilon << 1$. Within the limit of the first order of perturbation in ϵ the classical approach can be adopted for the present formulation as well. For the changes of the lift and surface pressure distribution, however, the details of the flow field around the airfoil are required. These can be calculated by satisfying the boundary condition on the airfoil surface. The change of the surface pressure can then be evaluated as

$$\Delta C_p(x) = \delta^{2/3} M^n (-2\phi_{1x}) \qquad \text{at } y = \pm 0 \tag{19}$$

The correction of lift is thus from Equation (16)

$$\Delta C_L = \delta^{2/3} M^n (2\Delta\Gamma) \tag{20}$$

The change of the pitching moment can be calculated from the surface pressure distribution, (Eq. (19)) as

$$\Delta C_M = \int_{-0.5}^{0.5} \Delta C_p(x)\,(x + 0.25)\,dx$$

referring to the quarter chord.

5. METHOD OF SOLUTION

The zero order equation and the conditions governing the free air flow have been extensively studied and computer codes have been developed for its calculation (Refs. 2, 9, 11). For the cases calculated in this report, the zero order solution is generated by a computer code available in the Laboratory*. The first order equation can be solved in a similar manner as the zero order equation by a line relaxation method. However, since the first order equation is linear, the difference equation can be solved directly by an existing method for solution of a system of linear algebraic equations. Some details of the numerical procedure for the solution of the first order equation are given as follows:

Co-ordinate Transformation

The x-co-ordinate is transformed into ξ as

$$\xi = \frac{2}{\pi}\tan^{-1}\left(\frac{x}{s}\right) \tag{21}$$

such that

$$\xi = 0 \rightarrow \pm 1 \qquad \text{as} \qquad x = 0 \rightarrow \pm\infty$$

where s is a parameter for scaling x. With the transformation the infinite extent of the x axis is brought within a finite domain bounded by $\xi = \pm 1$ and $y = \pm H$. For finite difference formulation, the ξ-axis is divided into NX points and y-axis into NY points. A finite difference equation can then be written for each grid point inside the domain.

* The author wishes to thank D.J. Jones for providing the zero order solutions for cases calculated in this report and for stimulating discussion on the course of the program.

Finite Difference Equation

In deriving the finite difference equation for the first order problem, the flow field is divided into the subsonic, the supersonic, the parabolic and the shock regions according to the local property of the zero order solution. An appropriate finite difference formula is assigned to each region accordingly (Ref. 11).

The types of flow are defined as follows:

Subsonic

$$\phi^o_{xc} = \frac{\phi^o_{i+1,j} - \phi^o_{i-1,j}}{2\Delta\xi}\frac{d\xi}{dx} < \frac{K}{\gamma+1} \tag{22a}$$

Supersonic

$$\phi^o_{xb} = \frac{\phi^o_{i,j} - \phi^o_{i-2,j}}{2\Delta\xi}\frac{d\xi}{dx} > \frac{K}{\gamma+1} \tag{22b}$$

Parabolic

$$\phi^o_{xc} > \frac{K}{\gamma+1}, \qquad \phi^o_{xb} < \frac{K}{\gamma+1} \tag{22c}$$

Shock

$$\phi^o_{xc} < \frac{K}{\gamma+1}, \qquad \phi^o_{xb} > \frac{K}{\gamma+1} \tag{22d}$$

where the superscript o denotes the zero order solution. The finite difference equation can now be written for each grid point. In the subsonic region, a central difference formula is used because of the elliptical nature of the second order partial differential equation. In the supersonic region, a backward difference formula is employed for its hyperbolic nature. At the parabolic points, the operation is achieved by setting

$$K - (\gamma + 1)\,\phi_{ox} = 0$$

and at the shock points, the operator is equivalent to the sum of the operators of the subsonic and the supersonic types. These operators are of second order accuracy.

With these operators, the interference potential equation, Equation (11a) can be written in a general form for a grid point (i, j) as

$$G\,\phi_{i-2,j} + A\,\phi_{i-1,j} + B\,\phi_{i.j-1} + C\,\phi_{i,j} + D\,\phi_{i,j+1} + E\,\phi_{i-1,j} = F \tag{23}$$

The details of the coefficients for each type of the flow and the boundary conditions are given in the Appendix. For all grid points, Equation (23) forms a system of linear algebraic equations which can be written in a metric form as

$$MX = Y \tag{24}$$

where M is an N X N matrix of block tri-diagonal form

$$M = \begin{bmatrix} P_1 R_1 & & & & \\ Q_2 P_2 R_2 & & & & \\ \cdot & \cdot & \cdot & & \\ & \cdot & \cdot & \cdot & \\ & & \cdot & \cdot & \cdot \\ & & & \cdot & \cdot \; R_{N-1} \\ & & & & Q_{N-1} P_N \end{bmatrix} \tag{25}$$

and the vector X and Y can be written in partitioned form as

$$X = \begin{bmatrix} \underline{X}_1 \\ \underline{X}_2 \\ \cdot \\ \cdot \\ \cdot \\ \underline{X}_N \end{bmatrix} \qquad Y = \begin{bmatrix} \underline{Y}_1 \\ \underline{Y}_2 \\ \cdot \\ \cdot \\ \cdot \\ \underline{Y}_N \end{bmatrix} \tag{26}$$

These matrices can be further identified as

$$P_1 = \begin{bmatrix} C_{2,1} D_{2,1} & & & & \\ B_{2,2} C_{2,2} \cdot & & & & \\ \cdot & \cdot & \cdot & & \\ & \cdot & \cdot & \cdot & \\ & & \cdot & \cdot & \cdot \\ & & & \cdot & \cdot \; D_{2,NY-1} \\ & & & & B_{2,NY} C_{2,NY} \end{bmatrix} \quad Q_1 = \begin{bmatrix} A_{2,1} & & & & & \\ & \cdot & & & & \\ & & \cdot & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ & & & & & A_{2,NY} \end{bmatrix} \quad R_1 = \begin{bmatrix} E_{2,1} & & & & & \\ & \cdot & & & & \\ & & \cdot & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ & & & & & E_{2,NY} \end{bmatrix}$$

and so on. The vector X is the unknown ϕ and the vector Y the right-hand sides of the equation, F.

Numerical Solution

The flow field bounded by the wind tunnel walls, the upstream and the downstream infinities is divided into a rectangular grid, with the following specification.

$$NX = 1, \ldots, 41; \qquad \Delta\xi = 0.05$$

$$NY = 1, \ldots, 22; \qquad \frac{\Delta y}{H} = 0.04878$$

The airfoil is located at the middle of horizontal lines NY = 11 and 12. After the solution is obtained, the solution at the airfoil plane is calculated by extrapolation.

The values of the first order solution at each grid point are obtained by interpolating the known solution calculated by the existing code. The coefficients for the difference equation, Equation (23) are then evaluated. The matrix equation, Equation (24) is solved by a subroutine LEQTIB in the International Mathematical and Statistical Libraries (IMSL) (Ref. 13).

The calculated examples reported below were performed on an IBM 3032 computer. The computing time is about 5CPU seconds for each run.

6. RESULTS AND DISCUSSION

The present formulation of the interference problem bases explicitly on the assumption of small perturbation. As discussed in Section 4, it is required that the perturbation parameter $\epsilon = 1/H$ be much smaller than unity. However, for most model-tests in a wind tunnel, the value of the parameter ranges from 0.30 to 0.50 which is relatively large for a first order theory. In principle, higher order terms in the perturbation series should be included. However, by comparison with the results of the exact small disturbance solutions (with tunnel walls), it has been found that for practical purposes the first order correction is adequate.

The test case chosen for the present method has the following conditions,

$$\begin{aligned} &\text{Airfoil} && - \ \text{NACA 0012} \\ &M && = 0.8 \\ &H^*/c && = 2 \\ &P^*_{U,L} && = 0.47 \text{ (Ref. 14)}, 0.44 \text{ (Ref. 2)} \end{aligned}$$

This case has been calculated by Murman (Ref. 2) and Catherall (Ref. 14) using the small disturbance theory. These results on the lift correction, unfortunately do not agree with each other. Details of tunnel wall pressure distribution are also available for Catherall's solution (Ref. 15).

The results for the lift corrections are shown in Figure 2. The free air solutions for the present calculations are those of Jones (Ref. 12) which are in good agreement with Murman's results. The solution for the wind tunnel case are obtained by adding the first order solutions of the tunnel interference to the free air solutions. The results of this first order theory agree reasonably well with those of Murman's direct solutions, though the perturbation parameter c/H has a value of 0.5 which is rather large for the first order theory. Catherall's solutions both for the free air and the wind tunnel are higher than Murman's and the present results. The *correction* of Murman's wind tunnel results back to the free air using the subsonic linear theory are also shown in the Figure for comparison.

The pressure distributions on the tunnel walls are shown in Figure 3 with the results of Catherall (Ref. 15) and Mokry (Ref. 5) for comparison. The present results follow that of Catherall's closely. The linear subsonic theory, however, fails to predict the pressure peaks on both walls. It might be argued that away from the airfoil, the non-linear effect decays rapidly and the linear theory should represent the local flow reasonably well. This indeed is true if the tunnel wall is located far enough away from the model so that the asymptotic solution is sufficiently accurate there ($|y| \simeq 6$) (Ref. 9). For the present case, the location of the wall has a value of 0.88 in y which is far too small for the asymptotic solution to be valid. This can be seen by comparing the pressure fields at a distance H away from the airfoil, as generated by the transonic small disturbance solution and the asymptotic solution of the linear theory for the airfoil in free air (Fig. 4).

Since the computation grid is chosen for the calculation of the interference *correction* as described in Section 4, it is too coarse for the calculation of the detailed pressure distributions on the airfoil surface. Further refinement of the numerical calculation is needed if the surface condition of the airfoil is required.

Two more cases have been calculated to check against the linear theory. In one case the flow is completely subsonic and the other case, supercritical. The airfoil has a relatively thick transonic profile and the conditions for these two cases are listed as follows:

Airfoil	– 16% thick transonic profile		
M	= 0.599		
H^*/c	= 3		
P_u^*	= 1.50		
P_L^*	= 0.50		
α	= 1.63°	α	= 3.90°
C_L	= 0.282	C_L	= 0.6

The porosity factors of the upper and lower wall are chosen to give the best fit of the linear theory to the experimental pressure distributions at the walls (Ref. 5). The calculated results are shown in Figure 5 with those of the linear theory for comparison. For the low lift case, the flow field is completely subsonic, the present results are fairly close to those of the linear theory as expected. The wall pressure distributions are shown in Figure 6. The non-linear compressibility effect can be observed but is not too pronounced. At higher lift, the flow over the upper surface is supercritical and with a shock terminating the supersonic region. The present results give a slightly higher (7%) angle of attack correction and much larger (25%) correction on Mach number. The wall pressure distributions are given in Figure 7 in which the effect of the non-linear compressibility is significant. These cases again show that the angle of attack corrections obtained by the linear theory are reasonably close to the non-linear results. By examining the far field asymptotic solution of the transonic equation, Equation (13), one may observe that the non-linear term appears as an additional doublet. Thus the circulatory flow is very close to that of the linear theory. The displacement flow, however, will be affected by the non-linear compressibility. This may explain that the Mach number correction predicted by the linear theory has a much larger discrepancy than that of the angle of attack.

7. CONCLUSION

The transonic wall interference is considered as a perturbation to the basic flow around the airfoil model in free air. Based on the transonic small disturbance theory, the perturbation equation is derived from the non-linear transonic equation and is linear but with variable coefficients containing the non-linear solution of the basic flow. With the boundary conditions imposed at the tunnel wall, the equation is solved numerically by a direct matrix method. The solutions for lift versus angle of attach agree reasonably well with those directly calculated from the small disturbance equation. The calculated pressure distributions at the tunnel walls are also in good agreement with the exact results. The present method is convenient to use for practical wall interference calculations if the matching of the solution with the tunnel wall pressure data is required. In that case the searching routine will be solving a linear equation only which is much quicker and simpler than solving the complete non-linear equation.

In comparison with the linear subsonic theory which is commonly used for correcting wind tunnel data, the present results show that the angle of attack and the Mach number corrections obtained by the linear theory work fairly well even in the transonic range. The linear theory underestimates the corrections by a small amount, for the cases here presented, which is well within normal experimental accuracy.

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LIST OF SYMBOLS

C_L	lift coefficient
C_p	pressure coefficient
D	doublet strength
H	tunnel half height
K	transonic similarity parameter
M	free stream Mach number
P	porosity factor
s	scale for x-axis
x,y	Cartesian co-ordinates
α	angle of attack
γ	ratio of specific heats
Γ	circulation around airfoil
δ	maximum thickness of airfoil
ξ	transformed co-ordinate
ϕ	velocity potential

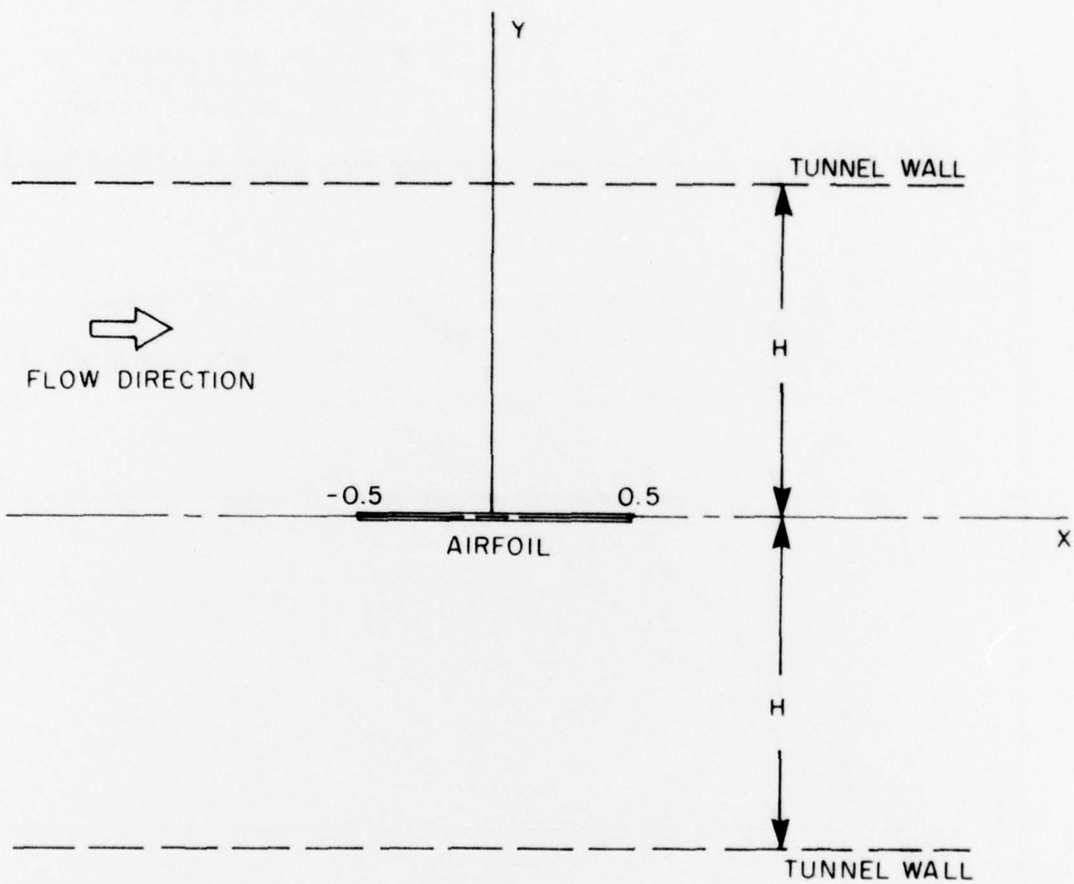

FIG. 1: PROBLEM LAYOUT AND CO-ORDINATES

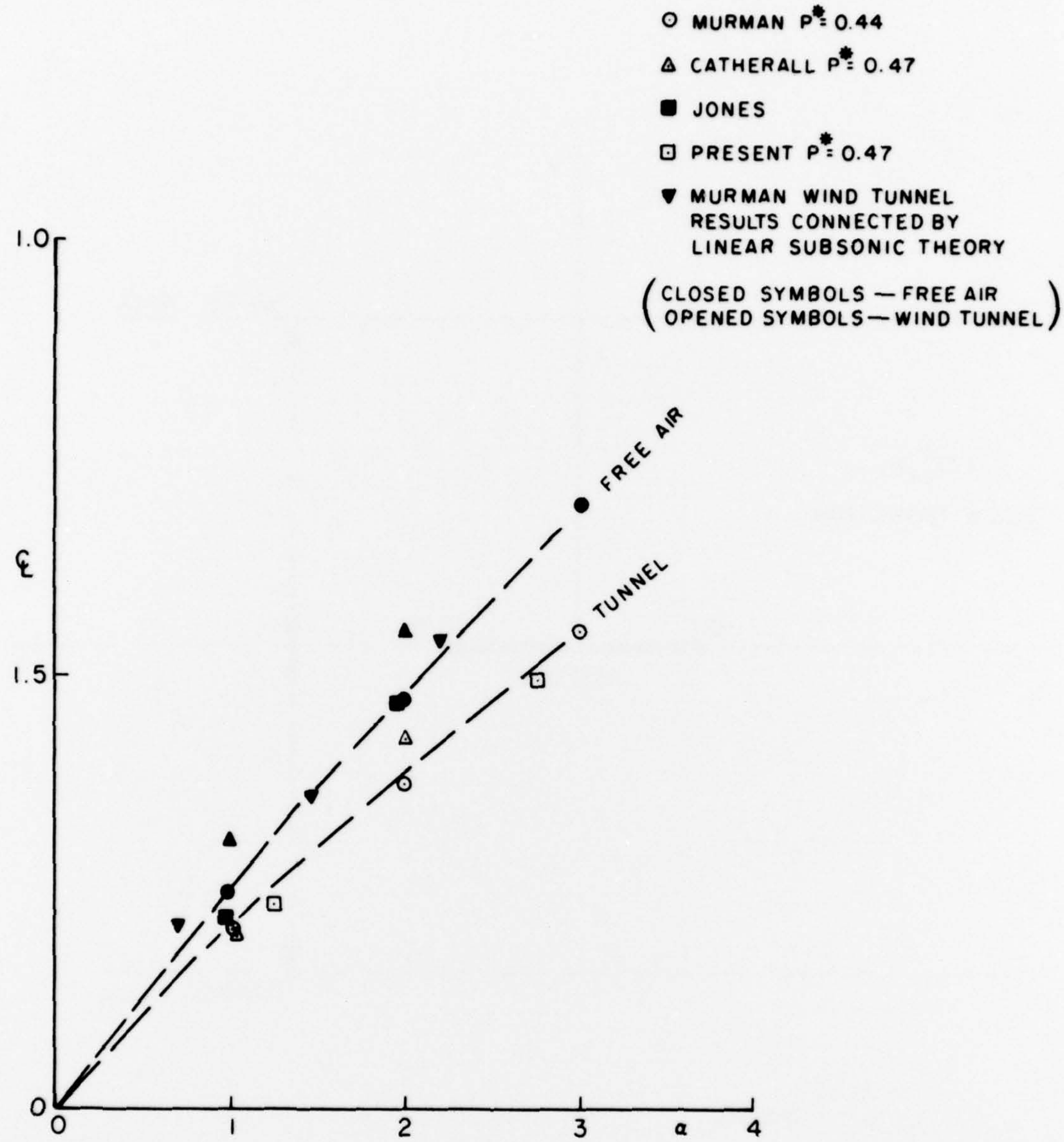

FIG. 2: COMPARISON OF DIFFERENT RESULTS FOR C_L vs α FOR NACA 0012 AIRFOIL IN PERFORATED WIND TUNNEL AND FREE AIR AT M = 0.80, H*/c = 2

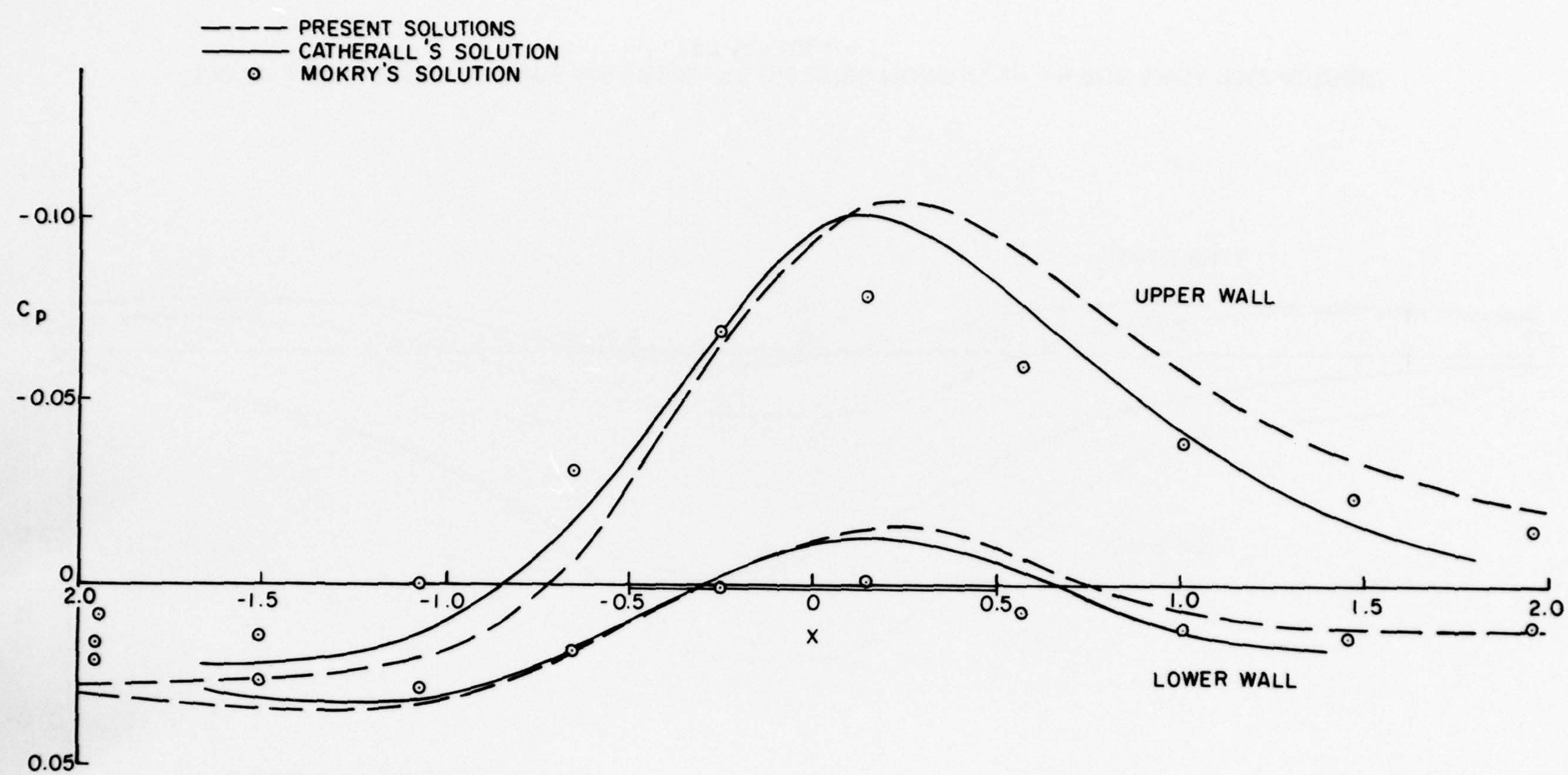

FIG. 3: COMPARISON OF PRESSURE DISTRIBUTIONS AT THE WIND TUNNEL WALLS FOR NACA 0012 AIRFOIL AT M = 0.8, H*/c = 2, P* = 0.47 and α = 1° (GEOMETRICAL)

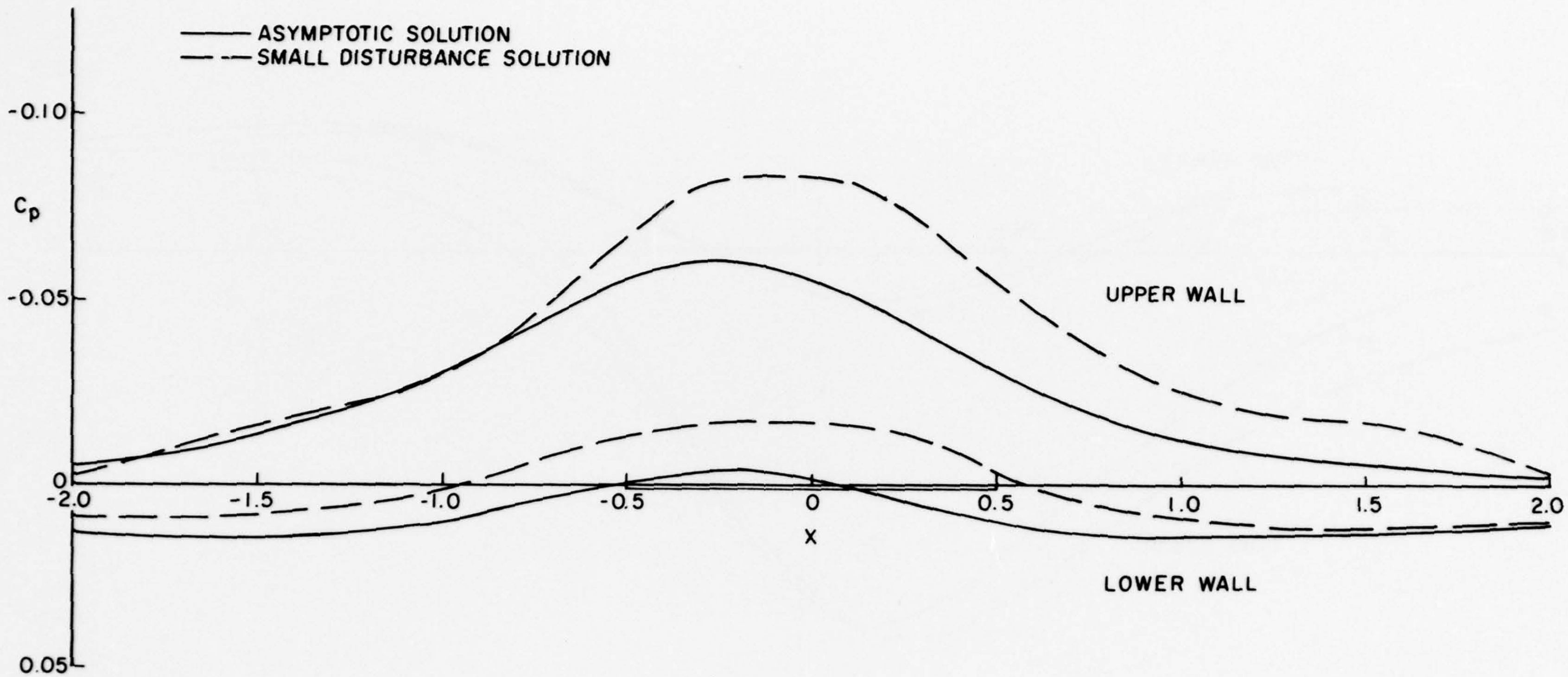

FIG. 4: COMPARISON OF FREE AIR PRESSURE DISTRIBUTIONS AT $y = \pm H$ FOR NACA 0012 AIRFOIL AT $M = 0.8$, $\alpha = 1°$

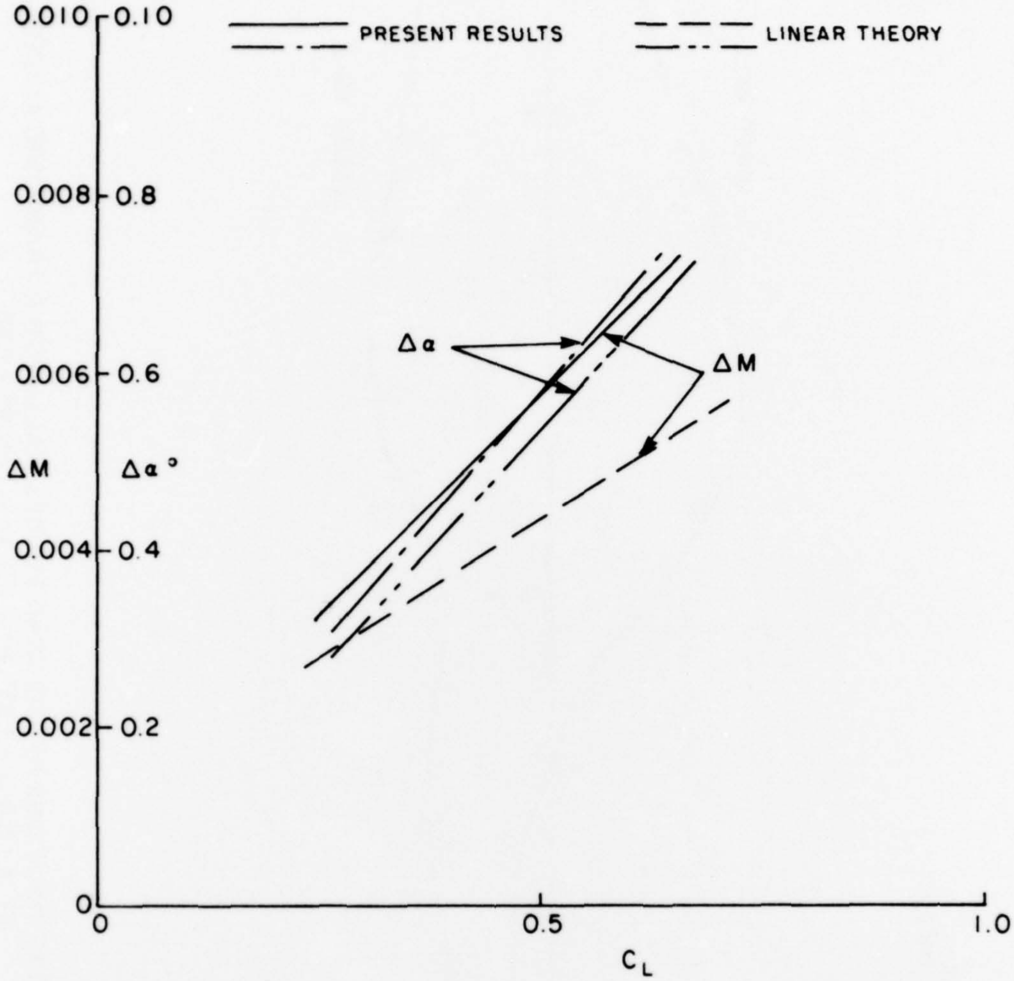

FIG. 5: CORRECTIONS FOR ANGLE OF ATTACK AND MACH NUMBER FROM PRESENT METHOD AND THE LINEAR THEORY

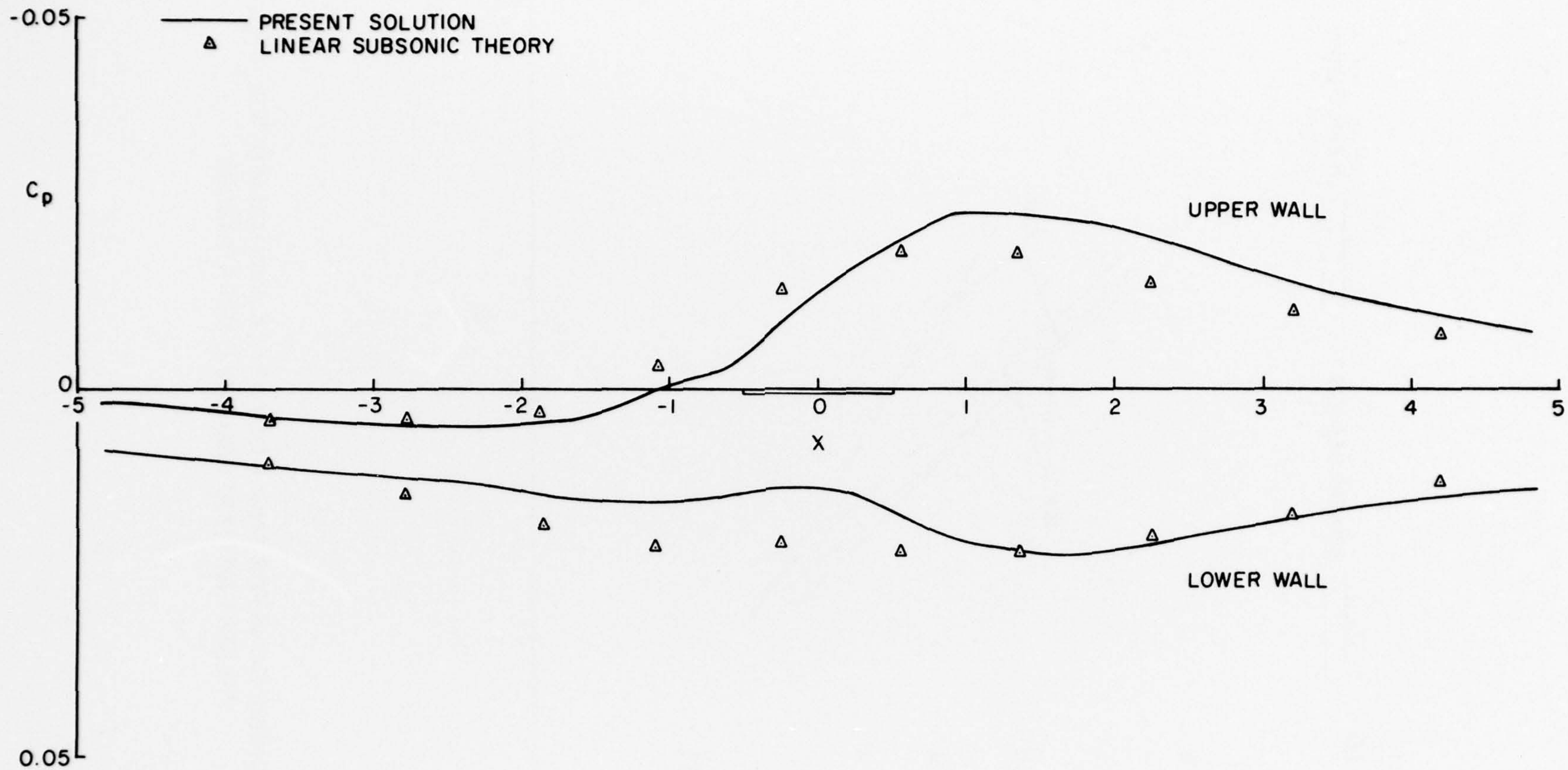

FIG. 6: PRESSURE DISTRIBUTIONS AT THE WIND TUNNEL WALLS FOR A TRANSONIC AIRFOIL AT $M = 0.6$, $\alpha = 1.63°$, $H^*/c = 3$, $P^*_u = 1.50$, $P^*_L = 0.50$

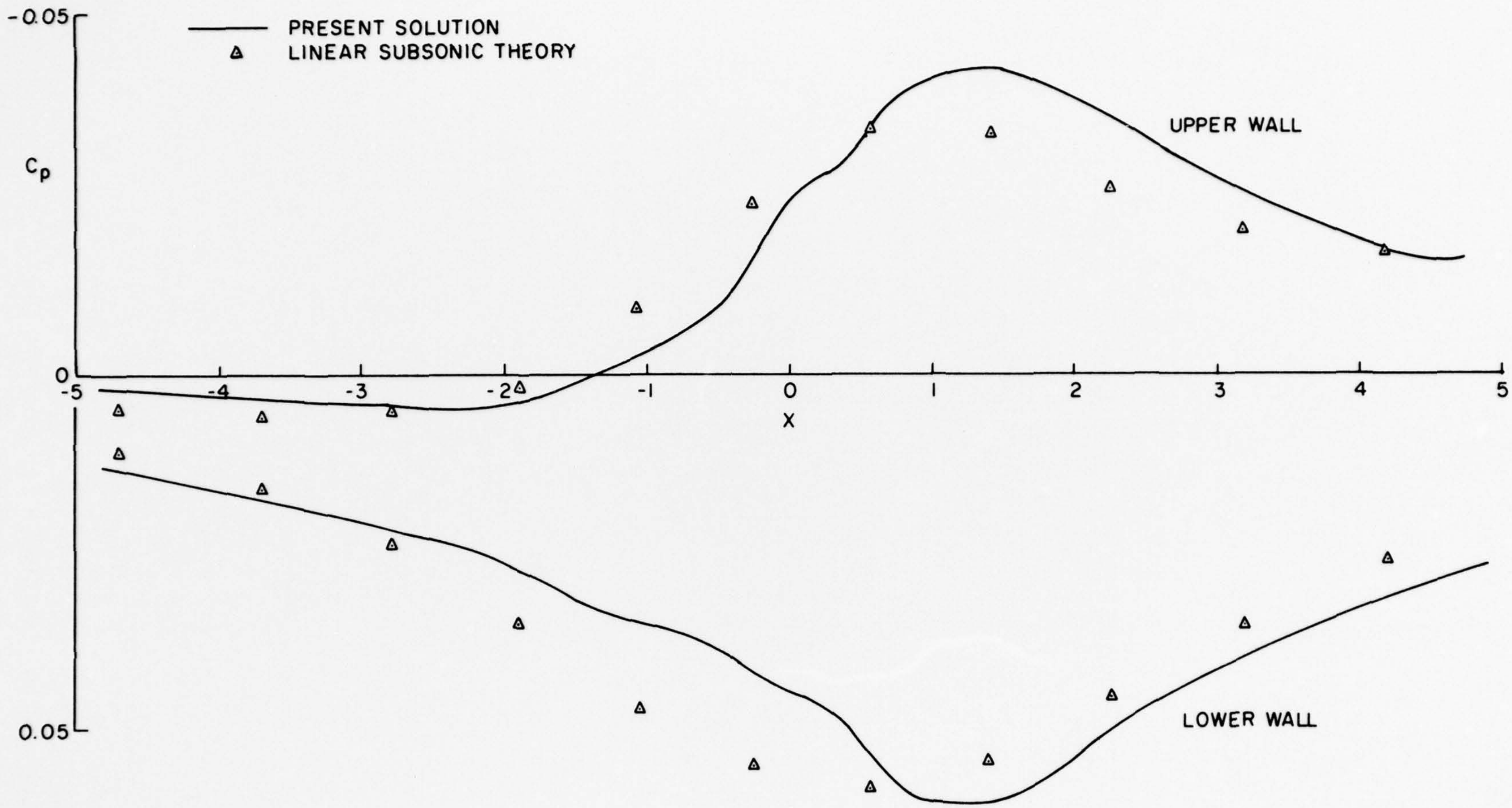

FIG. 7: PRESSURE DISTRIBUTION AT THE WIND TUNNEL WALLS FOR A TRANSONIC AIRFOIL AT $M = 0.6$, $\alpha = 3.90°$, $H^*/c = 3$, $P_u^* = 1.50$, $P_L^* = 0.50$

APPENDIX

COEFFICIENTS OF THE FINITE DIFFERENCE EQUATION, EQUATION (23)

The coefficients of the finite difference equation, Equation (23) are derived as follows for each type of flow. The subscript 1 denoting the first order potential has been dropped.

Subsonic

$$A\,\phi_{i-1,j} + B\,\phi_{i,j-1} + C\,\phi_{i,j} + D\,\phi_{i,j+1} + E\,\phi_{i+1,j} = 0 \tag{A1}$$

where

$$A = \left[K - (\gamma+1)\,\phi_{ox}\right]\left(\frac{Z_2}{\Delta\xi^2} + \frac{Z_1 Z_3}{2\Delta\xi}\right) + (\gamma+1)\,\phi_{oxx}\,\frac{Z_1}{2\Delta\xi}$$

$$B = \frac{1}{\Delta y^2}$$

$$C = -2\left\{\left[K - (\gamma+1)\,\phi_{ox}\right]\frac{Z_2}{\Delta\xi^2} + \frac{1}{\Delta y^2}\right\}$$

$$D = \frac{1}{\Delta y^2}$$

$$E = \left[K - (\gamma+1)\,\phi_{ox}\right]\left(\frac{Z_2}{\Delta\xi^2} - \frac{Z_1 Z_3}{2\Delta\xi}\right) - (\gamma+1)\,\phi_{oxx}\,\frac{Z_1}{2\Delta\xi}$$

with

$$Z_1 = \frac{1}{s}\left[\frac{2}{\pi}\cos^2\left(\frac{\pi}{2}\,\xi\right)\right]$$

$$Z_2 = \frac{1}{s^2}\left[\frac{2}{\pi}\cos^2\left(\frac{\pi}{2}\,\xi\right)\right]^2$$

$$Z^3 = \frac{1}{s}\sin(\bar{a}\,\xi)$$

The coefficients A, B, C, D and E are evaluated at the grid point (i, j) and the functions Z_1, Z_2 and Z_3 are introduced because of the co-ordinate transform Equation (21). $\Delta\xi$ and Δy are the grid sizes along the ξ and y direction respectively.

Supersonic

$$G\,\phi_{i-2,j} + A\,\phi_{i-1,j} + B\,\phi_{9,j-1} + C\,\phi_{i,j} + D\,\phi_{i,j+1} = 0 \tag{A2}$$

where

$$G = \left[K - (\gamma+1)\,\phi_{ox}\right]\left(\frac{Z_2}{\Delta\xi^2} + \frac{Z_1 Z_3}{2\Delta\xi}\right) + (\gamma+1)\,\phi_{oxx}\,\frac{Z_1}{2\Delta\xi}$$

$$A = -\left[K - (\gamma+1)\,\phi_{ox}\right]\frac{2Z_2}{\Delta\xi}$$

$$B = \frac{1}{\Delta y^2}$$

$$C = \left[K - (\gamma + 1)\,\phi_{ox}\right]\left(\frac{Z_2}{\Delta\xi^2} - \frac{Z_1 Z_3}{2\Delta\xi}\right) - (\gamma + 1)\,\phi_{oxx}\,\frac{Z_1}{2\Delta\xi} - \frac{2}{\Delta y^2}$$

$$D = \frac{1}{\Delta y^2}$$

Parabolic

$$A\,\phi_{i-1,j} + B\,\phi_{i,j-1} + C\,\phi_{i,j} + D\,\phi_{i,j+1} + E\,\phi_{i+1,j} = 0 \qquad \text{(A3)}$$

where

$$A = (\gamma + 1)\,\phi_{oxx}\,\frac{Z_1}{2\Delta\xi}$$

$$B = \frac{1}{\Delta y^2}$$

$$C = -\frac{2}{\Delta y^2}$$

$$D = \frac{1}{\Delta y^2}$$

$$E = -(\gamma + 1)\,\phi_{oxx}\,\frac{Z_1}{2\Delta\xi}$$

Shock

$$G\,\phi_{i-2,j} + A\,\phi_{i-1,j} + B\,\phi_{i,j-1} + C\,\phi_{i,j} + D\,\phi_{i,j+1} + E\,\phi_{i+1,j} = 0 \qquad \text{(A4)}$$

where

$$G = \left[K - (\gamma + 1)\,\phi_{ox}\right]\left(\frac{Z_2}{\Delta\xi^2} + \frac{Z_1 Z_3}{2\Delta\xi}\right) + (\gamma + 1)\,\phi_{oxx}\,\frac{Z_1}{2\Delta\xi}$$

$$A = \left[K - (\gamma + 1)\,\phi_{ox}\right]\left(\frac{-Z_2}{\Delta\xi^2} + \frac{Z_1 Z_3}{2\Delta\xi}\right) + (\gamma + 1)\,\phi_{oxx}\,\frac{Z_1}{2\Delta\xi}$$

$$B = \frac{1}{\Delta y^2}$$

$$C = -\left[K - (\gamma + 1)\,\phi_{ox}\right]\left(\frac{Z_2}{\Delta\xi^2} - \frac{Z_1 Z_3}{2\Delta\xi}\right) - (\gamma + 1)\,\phi_{oxx}\,\frac{Z_1}{2\Delta\xi} + \frac{2}{\Delta y^2}$$

$$D = \frac{1}{\Delta y^2}$$

$$E = \left[K - (\gamma + 1)\,\phi_{ox}\right]\left(\frac{Z_2}{\Delta\xi^2} - \frac{Z_1 Z_3}{2\Delta\xi}\right) - (\gamma + 1)\,\phi_{ox}\,\frac{Z_1}{2\Delta\xi}$$

At the wind tunnel walls, the boundary conditions Equation (11c) have to be satisfied. Along the lower wall, at grid points (i,j=1), the value of $\phi_{i,j-1}$ is replaced by the boundary condition. Assuming the flow is subsonic there, the equation becomes

$$A\,\phi_{i-1,j} + C\,\phi_{i,j} + D\,\phi_{i,j+1} + E\,\phi_{i+1,j} = F \qquad \text{at } y = -H,\ j = 1 \tag{A5}$$

$$A = \left[K - (\gamma + 1)\,\phi_{ox}\right]\left(\frac{Z_2}{\Delta\xi^2} + \frac{Z_1 Z_2}{2\Delta\xi}\right) + (\gamma + 1)\,\phi_{oxx}\,\frac{Z_1}{2\Delta\xi} + \frac{P_L Z_1}{\Delta y \Delta\xi}$$

$$B = 0$$

$$C = -2\left\{\left[K - (\gamma + 1)\,\phi_{ox}\right]\frac{Z_2}{\Delta\xi^2} + \frac{1}{\Delta y^2}\right\}$$

$$D = \frac{2}{\Delta y^2}$$

$$E = \left[K - (\gamma + 1)\,\phi_{ox}\right]\left(\frac{Z_2}{\Delta\xi^2} - \frac{Z_1 Z_2}{2\Delta\xi}\right) - (\gamma + 1)\,\phi_{oxx}\,\frac{Z_1}{2\Delta\xi} - \frac{P_L Z_1}{\Delta y \Delta\xi}$$

$$F = \frac{2P_L}{\Delta y}\left(\phi_{ox} - \frac{1}{P_L}\,\phi_{oy}\right)$$

P_L is the porosity factor of the lower wall.

Along the upper wall, at grid points (i,j=NY), the value of $\phi_{i,j+1}$ is eliminated by the boundary condition. The equation becomes

$$A\,\phi_{i-1,j} + B\,\phi_{i,j-1} + C\,\phi_{i,j} + E\,\phi_{i+1,j} = F \qquad \text{at } y = +H,\ j = NY \tag{A6}$$

where

$$A = \left[K - (\gamma + 1)\,\phi_{ox}\right]\left(\frac{Z_2}{\Delta\xi^2} + \frac{Z_1 Z_3}{2\Delta\xi}\right) + (\gamma + 1)\,\phi_{oxx}\,\frac{Z_1}{2\Delta\xi} + \frac{P_u Z_1}{\Delta y \Delta\xi}$$

$$B = \frac{2}{\Delta y^2}$$

$$C = -2\left\{\left[K - (\gamma + 1)\,\phi_{ox}\right]\frac{Z_2}{\Delta\xi^2} + \frac{1}{\Delta y^2}\right\}$$

$$D = 0$$

$$E = \left[K - (\gamma + 1)\,\phi_{ox}\right]\left(\frac{Z_2}{\Delta\xi^2} - \frac{Z_1 Z_3}{2\Delta\xi}\right) - (\gamma + 1)\,\phi_{oxx}\,\frac{Z_1}{2\Delta\xi} - \frac{P_u Z_1}{\Delta y \Delta\xi}$$

$$F = \frac{2P_u}{\Delta y}\left(\phi_{ox} + \frac{1}{P_u}\,\phi_{oy}\right)$$

P_u is the porosity factor of the upper wall.

At upstream and downstream infinity of the flow, the boundary condition is given by Equation (15). Thus we set

$$\phi_{i,j} = -\phi_o \quad \text{at } x = \pm\infty \text{ or } i = 1, NX \qquad j = 1, 2, \ldots, NY. \tag{A7}$$

In the classical approach of tunnel wall interference, the interference potential is calculated with the airfoil shrunk to a singularity point. The boundary condition in the airfoil, Equation (11b) need not be applied. However, if the details of the flow field around the airfoil are required, they can be calculated with the boundary condition on the airfoil enforced. The finite difference formula of the second derivative in the y-direction applying on the boundary condition can be written as (Ref. 8)

$$(\phi_{yy})_{i,j} = \frac{1}{\Delta y^2}\left[\phi_{i,j+1} - \phi_{i,j}\right] - \frac{(\phi_y)_{y=o}}{\Delta y} \tag{A8}$$

For the present condition in Equation (11b) the last term is zero. This condition should be applied at both the supper and the lower surface of the airfoil.